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ABSTRACT

The purpose of the study was to develop and validate a new method for measuring the subjective information of written materials in mathematics. In order to support the theoretical validity of the new measure, the study provided a derivation of the measure from Shannon's original information formula. Then the measure was used to measure the subjective information of a geometry passage for eighty-five seventh-grade students at various times during an instructional sequence in mathematics. The data obtained from these measurements was utilized to test the validity of the measure empirically and to investigate the information processing behaviors of the students. (Author/MN)

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Measuring the Information Content of Written Materials in Mathematics[1]

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[1]This paper represents a portion of the author's doctoral dissertation research conducted at Purdue University under the direction of Gerald Kulm.

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In order to study and assess learning and instruction within an information processing paradigm, the success of instruction must be evaluated and the psychological processes taking place during learning must be investigated. Traditionally, instructional evaluation procedures have employed achievement tests in their assessment operations. But achievement tests do not measure the actual amounts of information either contained in the instructional materials or learned by the students. They focus more on the product of instruction than on the psychological processes occurring in the learner (Glaser, 1976). For this reason, achievement tests seem inadequate as assessment tools in information processing investigations. A different type of measurement procedure seems to be required. The objectives of the present study were to develop and validate a new method for measuring the information content of written instructional material, and to investigate the method's effectiveness in assessing information transmission and processing in mathematics learning and instruction.

Background

In 1948 Shannon developed a probabilistic measure of the amount of information contained in a transmitted message. The values he obtained depended on the distributional statistics of the signs being transmitted. Since then, a number of adaptations of Shannon's original measure have been developed (Shannon, 1951; Newman and Gerstman, 1952; Taylor, 1955; Weltner, 1973), and a number of attempts have been made to investigate a variety of psychological constructs using concepts from information theory (Miller, 1956; Crossman, 1969). It was found that the human mind has a limited capacity to process stimuli, and that the limits could be stated in terms of bits of information.

One type of adaptation of Shannon's measure uses the "guessing-game" technique in which subjects guess each succeeding sign in a message. The data obtained is used to produce a measure of the subjective information of the message, the subjective information being the expected amount of uncertainty the receiver has about receiving the sequence of symbols occurring in the message. By its very construction then, the subjective information of a message depends on the relationship between the semantic and syntactic constraints of the message and certain characteristics of the receiver (e.g., age, aptitude, motivation, background, etc.). And because of this relationship, subjective information has potential as a variable for studying cognitive processes.

But although the concept of subjective information seems to have great promise as a tool for studying information processing behaviors, all the previously developed measures of subjective information seem inadequate. They were developed specifically to approximate Shannon's original concept of information, and his concept does not depend on characteristics of the receiver. Thus, previously developed information measures do not truly represent a receiver's uncertainty in receiving a particular message. The present study develops a new measure of subjective information that is intended to avoid the shortcomings of previous measures. The study then attempts to validate the measure with empirical data.

Description of the New Measure

The new formula for measuring the subjective information per token[2] in a

[2]In this study, "token" refers to word tokens and mathematics tokens. A word token is a unit having lexical meaning and separated from surrounding context by spaces. Mathematics tokens are signs such as +, $\surd$, +; letters of any alphabet when used as variables, and numerals.

written passage has the form

$$\sum_{i=1}^{k} P_i \; I(a_i).$$

It is applied to student responses on a guessing test over the passage in order to determine the amount of information the passage has for a group of students reading it. The new formula is derived from the original information formula developed by Shannon (1948).

The number k in the formula represents the total number of symbols in the passage. If the same token occurs in two different places, it is considered as two different symbols. Thus, k is the total number of tokens in the passage and P_i, the probability that token a_i occurs, is equal to $1/k$ for each token. To determine the information in token a_i, $I(a_i)$, the tokens of the passage are guessed one token at a time by a group of students. After each token is guessed, the correct token is revealed to the students. The probability that token a_i could have been guessed by the students in the group is taken to be $\alpha(n_i/n) + \beta$, where n_i is the number of students who correctly guessed token a_i, n is the total number of students in the group, and α and β are adjustment factors. The value of β represents the nonzero probability of token a_i being guessed by a member of the theoretical population when the sample value of n_i is equal to zero. The value of α is chosen so that $\alpha + \beta = 1$. The information in token a_i is then $I(a_i) = -\log_2[\alpha(n_i/n) + \beta]$.

The adjustment factors, α and β, are required because, without them, the expression for $I(a_i)$ would be equal to infinity when n_i is zero. Since it is unlikely that any token contains infinite information, the adjustment factors

were introduced so that if $n_i = 0$, the value of $I(a_i)$ would be $\log_2 1/\beta$. Thus, β is chosen so that $\log_2 1/\beta$ is an estimate of the amount of information contained in a token that no student in the population sample was able to predict correctly. In the present study, the value of β was taken to be 11 bits. This value seems in line with values obtained from studies of word frequency statistics (Newman and Gerstman, 1952; Miller, 1956; Hipkind and Nebonne, 1970).

If $n_i = 0$, then $I(a_i) = \log_2 1/\beta = 11$ bits, and so β is equal to $1/2{,}048$. Since α was chosen so that $\alpha + \beta = 1$, that makes α equal to $2{,}047/2{,}048$. So the complete formula for the subjective information per token is

$$H = -\frac{1}{k} \sum_{i=1}^{k} \log_2 \left[\left(\frac{2047}{2048} \right) \left(\frac{n_i}{n} \right) + \frac{1}{2048} \right],$$

where H is the average subjective information per token, k is the number of tokens in the passage, n is the number of students in the guessing group, and n_i is the number of students who guessed token a_i correctly. The units of the formula are "bits per token."

Problems and Hypotheses

In order to investigate the properties of the new subjective information measure and in order to assess the information processing behaviors of students during learning, the new formula was employed to measure information transmission in an educational setting. The following hypotheses were tested with data obtained with the new subjective information measure.

H_1: If two groups of students are approximately the same in previous learning and ability, then the amount of subjective information that a written passage has

for one group will be equal to the amount it has for the other group.

H_2: If students are given instruction over the material covered in a passage, the subjective information of the passage for the students will be lower after instruction than before instruction.

H_3: A passage will have less information for high ability students than for low ability students.

H_4: Given a group of students who must repeat an instructional unit, the decrease in the subjective information of a passage summarizing the content of instruction will be less during the second period of instruction than during the first.

H_5: Before instruction, the subjective information of a passage summarizing the content of the instruction will be less for students who will achieve mastery than for students who will not achieve mastery.

H_6: After instruction, the subjective information of a passage summarizing the content of the instruction will be less for students who achieved mastery than for students who did not achieve mastery.

In addition to the above hypotheses, two additional hypotheses were posed that are concerned with the following question: Are there differences in subjective information values for different types of textual material? This question is important because, the more sensitive is the measure in detecting differences in the information content of various types of text, the more promise it has for studying information processing of textual material.

H_7: A passage in mathematical English was prepared that consisted of four different types of material: the first type required the recall of definitions and attributes of some geometric terms, the second type required the recognition and naming of some geometric figures, the third type the calculation of perimeters of polygons, and the fourth type the estimation of the measures of angles. The style of language used for each type of material was characteristic of the type of material being discussed and was somewhat different for each type. In addition, the mental operations called for in the processing of the four types of material were different. It was therefore hypothesized that the subjective information per token would be different for the four types of material.

H_8: Mathematical symbolism is much more concise than ordinary English prose. Thus it was hypothesized that the subjective information per token would be greater for mathematical symbols than for ordinary English words.

Method

The subjects for the study were 85 seventh-grade students from a junior high school in a small rural midwestern community. The average IQ of the students was a little bit below average (96.34). This particular school was chosen for the experimental study because the instructional procedures used in the mathematics program at the school, as described below, seemed to provide an ideal situation for conducting an exploratory investigation with the new subjective information measure.

The instructional procedures that were followed during the experimental period were those normally used in the mathematics program of the school and are summarized as follows:

1. Students were placed in a unit if they had mastered the previous unit.
2. After two weeks of study, a unit average grade was determined for each student, with those achieving 80% or more advancing to another unit.
3. Students who did not achieve mastery repeated the unit during the next two weeks with a group studying the unit for the first time.

The two-week instructional units offered in the program covered the usual topics from seventh-grade mathematics. An introductory unit in plane geometry was chosen as the unit to be analyzed.

There were three groups of students participating in the study: Group R (repeaters) had taken the geometry unit in the previous two weeks and was repeating it, Group N (nonrepeaters) was studying the geometry unit for the first time, and Group C (control) was starting a pre-geometry unit on decimals.

A summary of the geometry unit was prepared which was 123 tokens in length. All three groups of students were given a guessing test over the summary passage both before and after the two-week instructional period. In addition, the test was given a third time to those students in Group N who did not advance to the next unit (Group NR) and to a sample of the students in Group C who did advance to the next unit (Group CN). The third test was given after the students in groups NR and CN had completed the subsequent offering of the original two-week geometry unit. The diagram summarizes the design of the study.

Group R	O_1	X_1	O_2		
Group N	O_1	X_1	O_2		
Group NR	O_1	X_1	O_2	X_1	O_3
Group C	O_1	X_2	O_2		
Group CN	O_1	X_2	O_2	X_1	O_3

Measurements O_1, O_2, and O_3 are the application of the same guessing test; X_1 is the two-week geometry unit; X_2 is the two-week decimal unit.

From the guessing tests, the subjective information of the passage was determined for each group for each time of testing. The values obtained were average information per token, H, and were calculated with the new formula. These values were then used to test hypotheses H_1, H_2, H_4, H_5, H_6, and H_7.

Hypotheses H_3 and H_8 were tested separately. Another mathematics passage (130 tokens) was prepared, and the subjective information of the passage was found for 42 students, 21 high ability and 21 low ability. (The average IQ of the high ability group was 101.68, and of the low ability group 90.13; the difference was significant at the .001 level.) Subjective information values for the entire group were used to test H_8; values from the two subgroups were used to test H_3.

Description of the Guessing Test

The guessing test was constructed by preparing an overhead transparency of the passage and covering every line of the transparency with a strip of masking tape. The transparency was then projected onto a screen. The experimenter instructed the students on how the guessing test was to proceed and the students guessed through a sample sentence to make sure that they understood the procedure. After the students wrote the first token of the test passage on their answer sheets, the experimenter pulled the masking tape back so that the first token was exposed. The subjects then guessed the second token. The procedure was continued until each word and mathematical symbol in the passage was guessed. The experimenter allowed about eight seconds for a token to be guessed unless a number of students asked for more time, in which case a few more seconds were given.

Scoring

Guesses for words or symbols were counted correct only if they were the exact word or symbol that appeared in the original passage. The only exception was that the word for a number was accepted for the corresponding numeral and vice versa. Misspelled words were counted correct if there was no ambiguity in identifying the word intended. Multiple digit numerals were guessed as whole entities; that is, "26", not "2" then "6".

Notes on the Use of Statistical Tests

Analysis of the effect of group size on information values revealed that as group size increased, subjective information values decreased. Thus, all comparisons of subjective information values for groups had to be based on groups of equal sizes. For example, in hypothesis H_1 the size of Group N was 23 and the size of Group C was 40. The information per word for Group N could not be compared to that of Group C directly; a sample size of 23 from Group C was needed. In order to obtain such a sample but to minimize further sampling error that would be introduced by taking just one sample, 10 random smaples of size 23 were taken from Group C. For every token in the passage, the information of the token for each sample was calculated and these ten values were averaged. The mean of these average values was then taken over all tokens in the passage. This derived mean information value for Group C was then comparable to the original mean information value obtained for Group N. The above procedure is an appropriate method for obtaining comparable subjective information values (Winer, 1976).

In hypotheses H_1, H_2, H_4, H_5, and H_6 the mean information per token of a passage for one group of students was compared to the mean information per token of the same passage for a different group of students. The means were taken as tokens varied, not as students varied, and the groups being compared were matched. Since it was found that the distribution of information values of tokens for a fixed group is nonnormal, a distribution-free test was required. The nonparametric Wilcoxon matched-pairs signed-ranks test was used for all comparisons of mean information per token for groups.

A non-parametric test, the Kruskal-Wallis ANOV, was used to test H_7. The t approximation to the randomization test was used to test H_8. See Siegel (1956) for the details of these tests.

Results

The information values related to hypotheses H_1, H_2, H_4, H_5, and H_6 are presented in Table 1. Comparisons of information values of these hypotheses with the Wilcoxon matched-pairs signed-ranks test are displayed in Table 2. The notation $HO_i(K)$ refers to the average information per token calculated from guessing test O_i when given to Group K.

For hypothesis H_1, the subjective information of the summary passage was found for two equivalent groups of students, Group N and Group C, before the first instructional unit began. The two groups were judged to be equivalent in previous learning and experience because Group N was only two weeks ahead of Group C in the instructional sequence. The equivalence was further supported by examining the students' IQ scores. The mean IQ for Group C was 94.19, and for Group N, 99.68. The IQ scores were not significantly different at the .05 level ($df = 63$, $t = 1.97$). The subjective information values obtained, based

Table 1

Mean Information Values per Token for All Groups[a]

(in bits)

Group	Test O_1		Test O_2		Test O_3		Differences	
	n	HO_1	n	HO_2	n	HO_3	$HO_1 - HO_2$	$HO_2 - HO_3$
N	23	2.81	23	1.02	--	--	1.79	--
C	23	2.57	23	1.86	--	--	.71	--
R	11	1.69	--	--	--	--	--	--
N[b]	11	3.34	--	--	--	--	--	--
R[c]	10	1.74	10	1.13	--	--	.61	--
N[d]	10	3.37	10	1.39	--	--	1.98	--
C[e]	10	3.29	10	2.39	--	--	.90	--
NR	6	3.93	6	2.31	6	1.52	1.62	.79
N-NR	6	4.20	6	1.37	--	--	2.83	--

[a] HO_i = information per token calculated from guessing test O_i.

[b] Based on samples of size 11 from Group N.

[c] Based on samples of size 10 from Group R.

[d] Based on samples of size 10 from Group N.

[e] Based on samples of size 10 from Group C.

Table 2

Wilcoxon Tests of Mean Differences of Information

Hypothesis	Size of guessing group	High value[a]	Low value[a]	N(total)	N(signed)	Wilcoxon T
H_1	23	HO_1(N)	HO_1(C)	123	119	3111.00
H_2	11	HO_1(N)	HO_1(R)	123	119	813.00***
H_2	23	HO_1(N)-HO_2(N)	HO_1(C)-HO_2(C)	123	122	1965.00***
H_4	10	HO_1(N)-HO_2(N)	HO_1(R)-HO_2(R)	123	121	1731.00***
H_4	6	HO_1(NR)-HO_2(NR)	HO_2(NR)-HO_3(NR)	123	109	1944.50***
H_5	6	HO_1(N-NR)	HO_1(NR)	123	107	2877.50
H_6	6	HO_2(NR)	HO_2(N-NR)	123	112	1395.00***
H_3	21	H for low ability	H for high ability	130	118	2109.50***

***p < .001

[a]"Value" refers to the sum of ranked differences of token information scores for the given group, (see Siegel, 1956).

on a sample size of 23 students per group, were 2.81 bits per token for Group N and 2.57 bits per token for Group C. Using the Wilcoxon test, it was found that the probability that the two samples came from the same population was .224. So the evidence supports hypothesis H_1 that the passage had the same subjective information for both groups.

There were two tests of hypothesis H_2 that the subjective information of a passage decreases when students are taught the material covered in the passage. In the first test, the subjective information of the summary passage for the repeaters, who had had geometry instruction, was 1.69 bits, and for the non-repeaters, who had not had geometry instruction, 3.34 bits, (see Table 1). The first Wilcoxon test displayed for hypothesis H_2 in Table 2 shows that this difference was significant at the .001 level. In the second test the decrease in the subjective information of the passage for students taught geometry for the first time, $HO_1(N) - HO_2(N)$, was found to be significantly greater ($p < .001$) than the decrease in subjective information for students taught decimals, $HO_1(C) - HO_2(C)$, (see Table 2). Since the only difference between the groups was that Group N received geometry instruction and Group C did not, it can be concluded that the geometry instruction reduced the subjective information of the passage for Group N.

For the test of hypothesis H_3, the mean information per token of the second passage for the high ability group was 3.43 bits and for the low ability group 3.79 bits. The Wilcoxon test showed that the H value for the high ability group was significantly lower ($p < .001$) than the H value for the low ability group.

Hypothesis H_4, that the decrease in subjective information of the summary passage would be less during the second period of geometry instruction than during the first, was supported by two separate results. First, as can be seen from Table 1, the decrease occurring for the repeaters was .61 bits, and the decrease for the nonrepeaters, 1.98 bits. The first Wilcoxon test displayed for hypothesis H_4 in Table 2 shows that this difference was significant at the .001 level. Second, for those students in Group N who had to repeat geometry instruction (Group NR), the decrease in the subjective information of the passage during the first period of instruction, $HO_1(NR) - HO_2(NR)$, and the decrease during the second period of instruction, $HO_2(NR) - HO_3(NR)$, were calculated (see Table 1). The second Wilcoxon test displayed for hypothesis H_4 in Table 2 shows that the decrease during the second period of instruction was significantly less ($p < .001$) than during the first.

Hypothesis H_5 predicted that the subjective information of the summary passage would be less for students who would achieve mastery on the geometry unit (Group (N-NR)) than for students who would not achieve mastery (Group NR). The information values for these groups displayed in Table 1, $HO_1(NR)$ and $HO_1(N\text{-}NR)$, show the opposite result, but the difference was not significant (see Table 2).

It was predicted in hypothesis H_6 that, after geometry instruction, the subjective information of the summary passage would be less for students who achieved mastery than for those who did not. As can be seen from Table 1, after instruction, the subjective information for the students who achieved mastery, $HO_2(N\text{-}NR)$, was less than the subjective information for students who did not achieve mastery, $HO_2(NR)$. The difference was significant at the .001 level (see Table 2).

It was predicted in hypothesis H_7 that the subjective information per token would be different for the four types of material contained in the geometry passage. The subjective information per token for the different types of text were: for definitions and attributes of geometric terms, 2.20 bits; for recognition and naming of geometric figures, 1.75 bits; for finding the perimeters of polygons, 2.83 bits; and for estimating angle size, 1.79 bits. The Kruskal-Wallis ANOV value of H for this data was 13.87, significant at the .001 level

Finally, it was predicted in hypothesis H_8 that the subjective information per token would be greater for mathematical symbols than for ordinary English words. The mean information per word was 3.44 bits; the mean information per symbol was 2.29 bits. The value of $\underline{t}$ for the $\underline{t}$ approximation to the randomization test was 2.30. The degrees of freedom for this $\underline{t}$ is 128; there were 82 words and 48 symbols. The $\underline{t}$ value is significant at the .05 level.

Discussion

Subjective Information and Knowledge

Several results of this study are relevant to the relationship between subjective information and knowledge. The results for hypothesis H_1 showed that if two groups of students are equivalent in previous learning and ability, the subjective information of a written passage will be the same for both groups. The results for hypothesis H_2 showed that the subjective information of a passage will decrease as knowledge of the passage or content of the passage increases. Finally, the results for hypothesis H_6 showed that, after instruction, the subjective information of a passage summarizing the content of instruction will be less for students who achieve mastery than for those who do not. Thus, it seems

that measuring the subjective information of a passage for students is a viable method for appraising the students' knowledge of the content of the passage. A decrease in the subjective information of a passage for students indicates that the student learning has occurred with respect to the passage.

One caveat must be offered though with respect to interpreting and utilizing subjective information values as a measure of knowledge. The subjective information of a passage for its recipient is the total uncertainty the passage has for the recipient. Included in this uncertainty is uncertainty about the ideas expressed in the passage, and uncertainty about the particular wording and phrasing used in the passage. Weltner (1973) refers to the former type of uncertainty as semantic information and the latter type as aesthetic information. So, when using subjective information values as a measure of subject knowledge, it must be realized that low values can indicate the presence of semantic knowledge, aesthetic knowledge, or both. Interpretation of such values must be made accordingly.

The fact that subjective information has both aesthetic and semantic components may seem to weaken its usefulness as a measure of comprehension and knowledge. But in fact, the componential nature of subjective information is one of its major assets as a tool for understanding the factors that play a role in the comprehension of written materials. Weltner (1973) has devised a method for resolving subjective information into its aesthetic and semantic components. He defined the aesthetic information of a passage as the subjective information of the passage for a group of students who have full knowledge and comprehension of the content of the passage. The semantic information of the passage is then the

total subjective information minus the aesthetic information. Weltner's resolution method can be used to obtain a more detailed analysis of the types of information and knowledge students gain from written materials and instruction.

Another aspect of the relationship between knowledge and subjective information is the prediction of achievement from subjective information values. According to information processing models of learning, the more information a learner has to process, the longer it will take him to process it. Therefore, one explanation for the situation in which some students master a certain amount of instructional material in a given amount of time and other students, of basically the same educational background and ability, do not, is that the material holds more information for the non-mastery students. This explanation was tested in hypothesis H_5 but was not supported by the results. Thus, the alternative and more credible explanation is that the instructional material holds the same amount of information for both mastery and non-mastery students, but that the students who do not achieve mastery fail to process a sufficient amount of the material in the given amount of time.

Subjective Information and Ability

The results supported hypothesis H_3 that the subjective information of a geometry passage would be lower for high ability students than for low ability students. The rationale for this hypothesis was that since brighter, more knowledgeable subjects are better at recognizing patterns and have a larger knowledge base, they should be less uncertain about what they perceive. That is, brighter subjects should be better able to impose some sort of organization or structure on the events they perceive, transforming events that are ostensibly

unrelated into sets of interrelated occurrences. The ability of the brighter subjects to more efficiently organize and structure their perceptions should reduce the amount of uncertainty contained in the perceptions thus causing the environment to have less information for them than for less able subjects. The results of the present study support this reasoning in the special case of written materials in mathematics.

The Effect of Repeated Instruction

The results of both tests of hypothesis H_4 supported the conclusion that for those students repeating geometry instruction, the decrease in subjective information of the summary geometry passage was greater during the first period of instruction than during the second. This finding makes credible the hypothesis that on initial exposure to new material, a great deal of subject uncertainty concerning the material derives from the vocabulary and the particular phrasing of language used to present the material. Thus, one possible explanation for the results obtained in the present study is that much of the information gained during the first period of instruction had to do with the students becoming familiar with what topics were to be covered in the instructional unit (i.e., developing a "mental set" for the unit) and with the students becoming familiar with the vocabulary and characteristic phrasing of these topics.

It is conjectured that during the second period of instruction, the students payed less attention to vocabulary and phrasing and more attention to comprehension of ideas. The conjecture is supported by the experimenters' observation that when new concepts were introduced during instruction, the students who were repeating instruction seemed much more comfortable with the new terminology than students who were receiving instruction for the first time. The students repeating

instruction seemed to recognize many of the new terms and to comprehend the new concepts much more quickly than their classmates who were not repeating instruction.

Differences in Types of Textual Material

Different types of textual material may be written in different language styles and may require the reader to utilize different mental operations. The mathematics passage used in testing H_7 consisted of four types of text requiring four different types of mental activities of the reader. The first type required students to recall definitions and attributes of geometric terms, the second type required students to recognize and name geometric figures, the third type to calculate the perimeters of rectangles, and the fourth to estimate the measures of angles. The findings for hypothesis H_7 were that the different types of text had significantly different subjective information values. In addition, there were clear indications that the subjective information values of the text types could be consistently ordered. The text requiring recognition of figures and estimation of angle size had the least subjective information for the students, the text requiring recall of definitions and attributes of terms the next higher subjective information values, and the text requiring calculation of perimeters the highest information values.

In order to interpret this ordering of subjective information values, the mental behaviors required of the reader by the different types of text were rated for complexity. The complexity of a mental behavior required by a sample of text was judged by the complexity of the type of learning that must have occurred in the reader in order for the text to be fully comprehended. Using Gagné's (1965)

nomenclature, comprehension of the first type of text required "concept learning" to have occurred in the reader, comprehension of the second and fourth types required "multiple discrimination learning," and comprehension of the third type required "principle learning." According to Gagné, if these three types of learning are rated in increasing order of complexity, they are multiple discrimination learning first, concept learning second, and principle learning third. So, if the four types of text used in the passage are ranked in increasing order of the complexity of mental behavior required by the text, the ordering is recognition of figures and estimation of angle size first, recall of definitions and attributes of terms second, and calculation of perimeters third. This is the same ordering that occurred when the types of text were ranked in increasing order of subjective information values. Thus, the findings suggest that the greater the complexity of mental behavior required by the text, the greater the subjective information of the text.

It should be noted here that an alternative explanation for the ordering in the subjective information values of the different text types is that the ordering was simply a result of the style in which this particular passage was written and was independent of the content of the passage. But the finding that different types of text have different subjective information values does seem to have a precedent; Rudolph (1971) found that relative sequential constraint, a type of information measure, varied between topics in mathematics textbooks. The results of the present study extend Rudolph's findings by suggesting that the subjective information of textual material depends on the mental activity called for in the material. Much more research using larger samples of textual material is needed in order to determine whether this proposed extension can be substantiated.

Mathematical Symbols versus Words

The results for hypothesis H_8 showed that the mean information per token was greater for words than for mathematical symbols. This result may seem somewhat surprising, for one has the intuitive notion that since mathematical symbolism is used to express ideas as concisely as possible, symbols should have high information values. Furthermore, previous research has shown that the presence of symbols contributes considerably to the reading difficulty of mathematical English (Kulm, 1973), supporting the notion that passages heavily laden with mathematical symbols contain more information and are harder to process. But in fact, in much mathematical English, symbols occur in very constrained and redundant situations. For example, in passages where an author has been very illustrative, say be doing examples, or by using labeled figures, or by including all the steps in an algebraic manipulation, there is likely to be very much repetition and contraint on the symbols. The majority of the mathematical symbols in such passages actually have very low subjective information values. So, although the results of the present study seem to contradict intuition, they may be typical of much textual material in mathematical English.

More research is needed, then, in order to clarify the information value of symbols in mathematical English. The sample of text used in the present study was very small; larger samples of text must be considered before any firm conclusions can be reached. In addition, more analysis must be made of the information value of symbols in different contexts. For instance, in the passage used for the present study, the first section dealt with perimeter and the symbols were used in connection with labeled figures, a very constrained situation. The second section dealt with circumference and the constraint on the symbols used

in this section was much less. Accordingly, the information per symbol in the section on perimeter was 2.20 bits; the information per symbol for the symbols in the section on circumference was 5.40 bits. The information per word for the entire passage was 3.44 bits. So, in some contexts mathematical symbols carry more information than words, and in other contexts the reverse is true. Context, therefore, seems to be a critical factor in determining the information value of symbols in mathematical English. Future research in this area should attempt to elucidate the information role of symbols in mathematical English by investigating how the information value of symbols varies with context. This would be the first step in discovering the role symbols play in student comprehension of written mathematics.

Conclusion

By measuring the quantity of information in written materials and the quantity of information processed by students during instruction, a different perspective for assessment can be achieved that can add new insights to our knowledge of student information processing behaviors. The results of the present study add to our knowledge of how students process mathematics and illustrate how a subjective information measure can be utilized to assess mathematical learning. The novel approach used in the study seems to have promise for future research into how students learn mathematics.

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